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ON THE NATURE OF THE QUASI-STELLAR OBJECTS

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ABSTRACT

In this paper we discuss the origin of the quasi-stellar objects from two different points of view: (i) that they are objects at cosmological distances as has been commonly supposed, and (ii) that they are local objects situated at distance $\sim 1 - 10$ Mpc. In the introductory section the optical properties of the quasi-stellar objects are compared with the optical properties of galaxies associated with strong radio sources and Seyfert nuclei from both points of view. Section II is devoted to a discussion of (i) and on this basis it is argued that they are most probably the nuclei of galaxies which have reached a high density phase at which time the formation of massive objects and their subsequent evolution has occurred.

Apart from the suggestion of Terrell little attention has been paid until now to the possibility that they are local objects and so we have considered this in considerable detail. A plausible case can now be made for supposing that they are coherent objects which have been ejected at relativistic speeds from the nuclei of galaxies at times when they erupt to give rise to strong radio sources and other phenomena. On this basis a likely candidate to give rise to the objects in our vicinity is NGC 5128 which is a powerful radio source in which at least two outbursts appear to have occurred. In this case some objects with blue shifts may be present. The fraction of such objects and the solid angle about NGC 5128 in which they lie is given as a function of the distance of NGC 5128, average speed of ejection, and time which has elapsed since the explosion. Calculations are also made of the red-shift magnitude relation to be expected in the local theory and comparison is made with the relation in orthodox cosmology.

Some of the problems associated with the hypothesis that the objects lie at cosmological distances have arisen through the observation of Dent of the

radio emission from 3C 273B. This shows that at 8000 Mc/s the intensity has increased by about 40% over the last two years, and that the spectrum is flat over the range 200 Mc/s to 8000 Mc/s and may be flat over a wider range. It is shown that a model based on the synchrotron process is able to explain the form of the spectrum with the object at a cosmological distance provided that the following conditions prevail: (1) the electron distribution is optically thin; (2) the magnetic field intensity depends on distance r from the center with the form $H = H_0(a/r)^n$; (3) the electron energy spectrum is everywhere the same; (4) the energy density of the electron distribution is of the form $W = W_0(b/r)^m$; (5) $n + m = 3$. Even with this model there is considerable difficulty in explaining a flat spectrum beyond 10^4 Mc/s. The difficulties associated with the model are somewhat reduced in the local theory.

In the concluding sections a number of programs are outlined which may enable us finally to determine whether the objects are at cosmological distances or are local.

I. INTRODUCTION

Of the sample of rather more than 100 radio sources for which optical identifications have been made more than 30 have turned out to be associated with star-like objects. Because optical identifications are confined to sources for which good radio position measurements have been made, and because good position measurements so far exist only for the brighter sources, it was thought until recently that data concerning these star-like objects could only be accumulated rather slowly. However, Sandage (1965) has begun to identify star-like objects which are not associated with the brighter radio sources. Sandage has described the objects as quasi-stellar galaxies, but in this paper we shall prefer the term quasi-stellar object since at this stage we do not think enough is known about their nature for a definitive name to be chosen. Indeed it is the purpose of this paper to discuss the ambiguities of interpretation of the quasi-stellar objects.

The new work of Sandage follows pioneering investigations by Humason, Zwicky, Haro, and Luyten on faint blue stars at high latitudes. It gives support to the view that quasi-stellar objects may be rather common - the at present rather poor statistics indicate as many as ~ 4 objects per square degree of sky, giving a total of $\sim 1.5 \times 10^5$. This implies that if the quasi-stellar objects are at cosmological distances their spatial density is about 0.01% of the density of galaxies. If the objects are closer than cosmological distances, their density is of course correspondingly greater. The new objects are radio quiet in the sense that if they are radio sources the flux at the Earth must be less than 10^{-25} W/m²/c/s at 178 Mc/s.

The quasi-stellar objects have been associated with galaxies for two reasons. Lines in their spectra show red-shifts similar to the cosmological

red-shift for galaxies, and those objects which are strong radio sources turn out to have intrinsic radio luminosities that are comparable with the intrinsic luminosities of strong radio galaxies - provided the red-shifts are interpreted cosmologically. However, both of these pieces of evidence are circumstantial. There is little or no direct evidence to connect quasi-stellar objects with galaxies. Indeed there is some evidence for an anti-correlation, for so far no quasi-stellar object has been found in a cluster of galaxies. The optical object associated with 3C 273 has an apparent magnitude of about 12.8. If 3C 273 were in a cluster of galaxies, the galaxies would have apparent magnitude about 18 and would easily be observed. The red-shifts for 3C 48, 3C 47, and 3C 147 are 0.367, 0.425, and 0.545, and at the cosmological distances indicated by these shifts galaxies would probably have been detected if these objects were in clusters. The shifts for 3C 254, 3C 245, CTA 102, 3C 287, and 3C 9 (Schmidt 1965) are so great that if any galaxies were associated with these objects they could be beyond the plate limit. Of the three new objects reported by Sandage, two have small red-shifts and associated galaxies, if there were any, would be readily observed. On the whole therefore the evidence is against quasi-stellar objects being correlated spatially with galaxies. It is to be anticipated that any remaining ambiguity in this question will soon be eliminated.

It is our purpose in this paper to discuss the further consequences of supposing

- (a) that the quasi-stellar objects are at cosmological distances,
- (b) that they are extragalactic but of local origin.

In Sections II and III we discuss the qualitative implications of (a) and (b) respectively, while in Sections IV and V we shall be concerned with more quantitative problems, in particular in Section IV with the question of whether

the observations of Dent (1965) of a variation in the intensity of the high frequency radio emission from 3C 273B throws light on the nature of this object.

II. QUASI-STELLAR OBJECTS AT COSMOLOGICAL DISTANCES

To avoid pedantry, we shall drop the conditional 'if at cosmological distances' clause and will write in this section as if the quasi-stellar objects were known to be at cosmological distances - the conditional clause obviously applies to the whole of this section. However, certain of the properties of quasi-stellar objects, briefly reviewed below, apply also to the conditions of Section III. These will be indicated by an asterisk. Following this review we shall consider three theories which have been proposed to explain the nature of the objects.

First, we note three properties which distinguish the quasi-stellar objects from normal galaxies.

- (1) Optically, the quasi-stellar objects are up to 400 times brighter than the most luminous galaxies. All of the quasi-stellar radio sources for which red-shifts are available have absolute magnitudes in the range -24 to -26, compared to -21 for the most luminous galaxies. Two of the three new objects observed by Sandage have small red-shifts and the absolute magnitudes are near -21. Hence the quasi-stellar objects cover a range extending upwards by 5 magnitudes from normal galaxies.
- (2)* The objects are all exceedingly compact. In the majority of cases they are indistinguishable from stars on direct plates, although in one of Sandage's new cases the object shows a fuzziness that distinguishes it from a stellar image.

On the cosmological hypothesis, this sets a limit to the size of the optical object at about 1 kpc for the nearer systems and about 3 kpc for the more distant objects.

A more powerful limitation on the sizes of the objects comes from variation in the optical brightness. In all cases in which repeated measurements have been made of the objects associated with radio sources variations in light have been seen or suspected. This means that a major part of the luminosity in such objects must come from a source that is at most only a few parsecs in extent.

(3)* The optical radiation emitted by these objects is not radiation from ordinary stars. The continuum is almost certainly a mixture of radiation from a hot diffuse gas with a non-thermal component, possibly synchrotron radiation. The emission lines are also those characteristic of a rather highly excited gas (Greenstein and Schmidt 1964; Schmidt 1964; Sandage 1965).

In contrast to these differences, certain interesting similarities emerge when quasi-stellar objects are compared with abnormal galaxies.

(4)* The emission lines in the spectra of quasi-stellar objects have widths which indicate large random motions in the emitting gas, ≥ 1000 km/sec. This is also a characteristic property of the spectra of the nuclei of Seyfert galaxies. In a general way, the latter also have the properties (3)*, although the emission lines are stronger relative to the continuum than is the case in quasi-stellar objects.

The nuclei of Seyfert galaxies are known to be compact. Their stellar appearance on direct plates sets a limit of about 50 pc to their sizes. Evidently, there is more than a superficial resemblance between such nuclei and the quasi-stellar objects. We can perhaps venture the predictions

(i) that the colors of these nuclei will turn out similar to the quasi-stellar objects,

(ii) that light variations will be found.

- (5)* A jet emerges from 3C 273B which is similar to the jet emerging from the nucleus of M 87.
- (6)* The radio source 3C 279 has two components separated by $\sim 20''$. The optical object associated with 3C 279 lies on the line connecting the two components (Veron 1965). This is similar to the situation in many radio galaxies. In this respect also the centers of galaxies appear to play the same role as quasi-stellar objects.
- (7) The linear size associated with 3C 47 is of order 200 kpc or more, which is comparable with the sizes of large radio galaxies.
- (8) The radio emission from the quasi-stellar sources is $\sim 10^{44}$ erg sec $^{-1}$, comparable with the strongest radio galaxies.
- (9)* The N-type radio galaxies have star-like nuclei outside which faint features can be seen (Matthews, Morgan, and Schmidt 1964). Like the fuzziness associated with 3C 48 these features could be due to a jet, or a series of jets, emerging from the nucleus. The compact galaxies described by Zwicky (1964) could be a similar phenomenon. Possibly Cygnus A should also be included in this category - i.e., of galaxies having features in common with quasi-stellar objects. The extent of the region giving emission lines in Cygnus A has dimensions of ~ 6 kpc, the absolute magnitude is about -21, and the emission lines in this system are very strong.

We are strongly impressed by items (4)* to (9)*, which seem to us to indicate a close connection between the physical processes in quasi-stellar objects and those which take place in the nuclei of some galaxies. It must be emphasized, however, that item (1) remains a major and critical difference. The absolute magnitudes of the nuclei of Seyfert galaxies are about -18, a thousand times fainter than the most luminous quasi-stellar objects. This difference is unavoidable so long as we accept the cosmological interpretation of the red-shifts.

We are then dealing with a phenomenon in which the optical output ranges from $\sim 10^{42}$ erg sec $^{-1}$ to $\sim 10^{46}$ erg sec $^{-1}$, and in which the emission comes from a volume of only a few cubic parsecs. This and the fact that there is no evidence of any stellar features in the optical spectra place important constraints on any theory which interprets the quasi-stellar redshifts as cosmological. If normal stars exist in the quasi-stellar objects they are completely overborne in luminosity by the continuum of thermal and non-thermal origin emitted by a hot plasma.

We turn now to three theories which have been put forward. Field (1964) has suggested that the objects are galaxies in the process of formation and Sandage (1965) has supported this point of view. The difficulty in this theory is to understand the similarities between the newly forming galaxies and the nuclei of old-established galaxies. Although we are not unsympathetic to the idea of newly forming galaxies (Burbidge, Burbidge, and Hoyle 1963), we have not so far been able to understand how items (4)* to (9)* can be understood in this theory.

A far more radical theory is that both the quasi-stellar objects and the nuclei of some galaxies are relics of a high density phase of the whole Universe. Many cosmologists are attracted by an oscillating model for the Universe. The critical problem in such a model is to explain why the Universe switches from contraction to expansion. In the model of Hoyle and Narlikar (1965a) the switch is explained with the aid of a new field, termed the C-field. So far no strict mathematical explanation has been given within the usual framework of cosmology; the switch is simply assumed. Granted, however, that a switch takes place in some fashion, it is reasonable to argue that if the whole Universe can 'bounce' so can a localized object. The time scale for the bounce of a localized object is not the same for a distant observer as it is for an observer moving with the

object. The former is greater than the latter by a dilatation factor, $(1 - \xi)^{-\frac{1}{2}}$, where ξ is the largest value of the relativistic parameter, $2 GM/R$ for a mass point in the usual Schwarzschild theory, attained during the oscillation. If ξ comes exceedingly near unity, as it can do in the theory of Hoyle and Narlikar, this dilatation factor can greatly exceed the oscillation time scale for a comoving observer. Indeed, the time scale for the external observer could be as long as 10^{10} years, so it would be possible for us to see localized objects emerging from a highly relativistic situation.

Ulam and Walden (1964), Gold (private communication), and Woltjer (1964) have all noticed that the star density is high near the centers of galaxies. They have suggested that star collisions may be frequent enough to produce appreciable optical emission. Acceleration of particles to cosmic ray energies in a rapidly moving gaseous assembly might be responsible for the radio emission. Similar ideas have also been suggested by these authors for the quasi-stellar objects.

The absence of any detectable stellar component in the spectra of the quasi-stellar objects casts doubt on this theory, at any rate on the idea that the optical emission arises from star collisions. It was also pointed out by Hoyle (1964) that the time variations in the optical emission of 3C 273 cannot be explained in terms of star collisions. It would seem, therefore, that a more hopeful line of attack would be to argue that star collisions are responsible for producing a massive object of the kind first discussed by Hoyle and Fowler (1963), and that both the optical and radio properties are controlled by the massive object. The further evolution of such an object has been studied by Hoyle and Fowler (1965) and by Fowler (1965).

This latter form of the third theory is perhaps the most conservative attack on the problem. The main difficulty in the theory is to understand how

the star density can become high enough to produce any appreciable development through the mechanism of star collisions. A qualitative discussion of this question has been given by Gold, Axford, and Ray (1965). Preliminary calculations by Ulam and Walden (1964) indicate that collisions will increase rather rapidly if the star density exceeds $10^6/\text{pc}^3$, and that the process becomes essentially catastrophic at a density of $10^9/\text{pc}^3$. These values may be compared with the star density at the center of M 31, $\sim 10^3/\text{pc}^3$. Nothing is known about the star density at the centers of other more distant galaxies.

The difficulty can be understood in more general terms in the following way. Divide the inner regions of a galaxy into a large number of small volumes, say the region within 100 parsec of the center into a million equal cells. Take the time average of the stars, and of their motions, within each cell - i.e., attach an observer to each cell and let him observe the stars that pass through his individual cell. Why should one particular observer, the one associated with a cell at the geometric center of the galaxy, obtain a result substantially different from any other observer? What distinguishes the center as a singular point? We believe these questions to be unanswerable, and the theory to be consequently untenable, if the nuclei of galaxies have condensed by contraction from a diffuse gas, in accordance with the usual picture of their origin. In such a picture we would expect the stars to have sufficient angular momentum about the geometric center for one cell in our imaginary model to be indistinguishable from another, at any rate over the first 10 to 100 parsecs from the center.

An alternative suggestion for the origin of the nuclear regions of galaxies, and for the elliptical galaxies as a whole, has recently been put forward by Hoyle and Narlikar (1965b). In their oscillating model the expansion phase takes place nearly as in the Einstein de Sitter cosmology. The latter is a

limiting case in the sense that comparatively small inhomogeneities can restrain expansion over limited volumes. It was suggested that elliptical galaxies are such restrained volumes possessing mass concentrations at their centers, the mass concentration at the center of a massive elliptical being $\sim 10^9 M_{\odot}$. On this basis it is possible to derive theoretically the form of the light distribution within ellipticals. This turns out in very good agreement with the observed distribution, suggesting that the expansion picture may well be correct. If so, the center is singular from the origin of a galaxy, it does not have to develop. The center can have an initial density comparable to the mean density of the Universe at its most compact state. The most suitable criterion for determining this density numerically is from a comparison with observation of the formula

$$2 \frac{Gm}{c^2} \approx c (G\rho)^{-\frac{1}{2}}, \quad (1)$$

where m is an upper limit to the total mass of clusters of galaxies. Here $(G\rho)^{-\frac{1}{2}}$ is the unit of time associated with the required mean density ρ . The right hand side of (1) is therefore of the order of the dimensions of a length, L say, and (1) expresses the theoretical result that the relativistic parameters associated with m and L , $2 G m / c^2 L$, must be of order unity. Numerically, (1) leads to

$$m \approx 4 \times 10^8 \rho^{\frac{1}{2}} M_{\odot}. \quad (2)$$

If we set m equal to the mass of a typical cluster of galaxies, $\sim 10^{13} M_{\odot}$, (2) gives $\rho \approx 10^{-9} \text{ gm cm}^{-3}$, which is close to the value used by Hoyle and Narlikar. However, it is possible that m should be set equal to the masses of the largest clusters, $\sim 10^{15} M_{\odot}$, in which case $\rho \approx 10^{-13} \text{ gm cm}^{-3}$. The latter value is close to what would be required to give $\sim 10^9$ stars per cubic

parsec. Although there would be some reduction of density due to expansion, an initial value equivalent to between 10^6 and 10^9 stars/ pc^3 is entirely possible.

On this basis elliptical galaxies, and perhaps some spirals, are born with their nuclei already at the critical density necessary for the development of massive objects. It is likely that violent events in which material is thrown out of the nucleus early in the history of such galaxies lead to a quasi-equilibrium in which the nucleus is always close to instability - i.e., to a further outburst. The occurrence of an outburst would be expected to stabilize the situation for a while, until further evolution eventually brings on a new outburst, or until the whole inner dense nucleus has been dissipated and the galaxy becomes finally inactive.

Of the three theories mentioned or discussed above the third seems to us in many ways the most attractive for the case of radio galaxies. For the quasi-stellar objects the third theory raises an awkward problem, however. Because of the similarities between quasi-stellar objects and radio galaxies we are loath to accept a quite different theory for the quasi-stellar objects. Yet if we suppose the latter to be massive objects situated at the centers of dense star systems we are obliged to ask what star systems, in particular what star systems can we have that are not associated with clusters of galaxies? A possible answer would be the dwarf elliptical galaxies which probably have a high spatial density, existing in profusion as field galaxies. The mystery then is why dwarf ellipticals can set up objects with a far greater optical output than the objects which develop at the centers of massive ellipticals. The natural expectation would be to have things the opposite way around.

III. QUASI-STELLAR OBJECTS AS LOCAL PHENOMENA

There is no question in our minds but that the line shifts which have been measured in the quasi-stellar radio sources and the objects studied by Sandage are Doppler shifts. The case against them being gravitational in origin has been made in detail for 3C 273 and 3C 48 by Greenstein and Schmidt (1964) and in our view it is overwhelming. If the objects are local it is therefore necessary to explain how velocities nearly up to c , relative to the usual standard of rest, have been derived. This is the immediate problem which any local theory has to face.

The minimum total energy necessary to explain the emission from strong radio galaxies is of order 10^{60} erg. This value is calculated on the basis of equipartition between the total energy of the magnetic field and the total energy of the synchrotron electrons, protons being assumed absent. Allowance for a deviation from equipartition and for protons making a dominant contribution to the energy could readily raise the requirement to 10^{62} erg. Hence we already know that $10^{60} - 10^{62}$ erg is involved in the outbursts of strong radio galaxies. The energy distribution of the relativistic particles, if it is at all like normal cosmic rays, is such that the main contribution to the total energy comes from particles with individual energies not much above 1 Bev $\approx 10^{-3}$ erg. It seems then as if we are involved in $10^{63} - 10^{65}$ particles moving at speeds comparable to c . This corresponds to a total mass of 10^6 to $10^8 M_{\odot}$ moving at relativistic speed. It is clearly permissible therefore to argue that a mass of the general order of $10^7 M_{\odot}$ is ejected at relativistic speed from a strong radio galaxy and it is on this that a local theory for the origin of the quasi-stellar objects must turn.

It has been customary to think of the matter ejected from radio galaxies as a diffuse cloud of separated particles. What we have now to consider is

the possibility that in addition to a diffuse emission there may also be an ejection of compact objects, and that such objects make up an appreciable fraction of the ejected material. In several respects it is easier to understand the observational data in these terms. If a single object breaks explosively in to two objects, the two objects must fly apart in opposite directions, agreeing immediately with the characteristic property of radio galaxies, that they tend to be double and that the join of the two sources tends to pass through the center of the associated galaxy. A phenomenon such as the jet of M 87 would seem to be more readily understood if a series of compact objects exists along the line of the jet. Otherwise it is hard to see why radial lines of force of a magnetic field should be confined to a jet. Also numerous condensation knots appear to exist within the jet.

Rather than suppose the nucleus of a galaxy to eject a large number of compact objects, it is possible that the number of objects grows by repeated subdivision. First there are two major objects, then each of these objects breaks in to two, and so on in a cascade process. The number counts which Sandage has made from the Haro-Luyten catalogue of blue 'stars' suggests the operation of some controlled break-up process. Write $N(m)$ for the number of objects brighter than magnitude m . Sandage finds $d \log N/dm = 0.383$. We obtain substantially this relation from the following postulates:

- (i) The objects have expanded out from a local source and are now approximately isotropically distributed with respect to the Galaxy.
- (ii) The total mass of the objects within unit logarithmic interval of mass is constant.
- (iii) Postulate (ii) applies not only to the total distribution of objects but at every ejection speed.
- (iv) The optical output of an object is proportional to its mass.

The second postulate requires the number of objects with masses between M and $M + dM$ to be proportional to dM/M^2 . Using (iv) the number with intrinsic luminosities between L and $L + dL$ is proportional to dL/L^2 . If all the objects are at the same distance, as those with a particular ejection speed are in view of (i), the number with apparent luminosities between S and $S + dS$ is proportional to dS/S^2 and the number brighter than S is proportional to $1/S$. If this is true for every ejection speed it is true for the total distribution of objects. Writing $N(S)$ for the number brighter than S ,

$$\log N = -\log S + \text{constant} = 0.4 m + \text{constant} \quad (2)$$

$$\text{and } d \log N / dm = 0.4.$$

The inference from the counts, that postulate (ii) may be true, has an important application, for it means that the total mass requirement is

$$\sim M_{\max} \ln M_{\max}/M_{\min}, \quad (3)$$

where M_{\max} is the maximum mass to be found among the objects, and M_{\min} is the minimum mass. Since the logarithmic factor is unlikely to be much greater than 10, the mass requirement is not more than $\sim 10 M_{\max}$, so that the most massive and brightest object can contain as much as ten percent of the total mass. With $\sim 10^7 M_{\odot}$ for the latter, we can have a maximum object mass of $\sim 10^6 M_{\odot}$. Since our distribution requires the number of objects more massive than M to be proportional to M_{\max} , we have

1 object with mass M_{\max} ,

10 objects with masses $> 0.1 M_{\max}$,

100 objects with masses $> 0.01 M_{\max}$,

and so on.

The advantage of the local theory is that it relates the properties of quasi-stellar objects immediately and directly to the radio galaxies. They are of the same stuff, with a similar structure, to the objects giving rise to the

properties of the radio galaxies. Similarities, such as items (4)* to (9)* of Section II, become much more readily understandable.

Two possibilities arise in the local theory for the source of the quasi-stellar objects. Terrell (1964) has suggested that the objects have been ejected from the nucleus of our own Galaxy and he has pointed out that Burbidge and Hoyle (1963) proposed an explosion in the galactic nucleus in order to explain the existence of a transient halo and the outflow of gas in the plane of the Galaxy. According to Burbidge and Hoyle the explosion occurred about 10 million years ago. Hence if we take $c/3$ as the characteristic ejection speed of the objects their present distances should be about 1 Mpc. Since the characteristic distance of the objects on the cosmological picture is $\sim 10^3$ Mpc, energy requirements are reduced by a factor $\sim 10^6$. The optical emission of 3C 273, instead of being $\sim 10^{46}$ erg sec $^{-1}$, becomes $\sim 10^{40}$ erg sec $^{-1}$, and the total emission over 10 million years is $\sim 10^{55}$ erg. Because 3C 273 is probably one of the brightest of the quasi-stellar objects (intrinsically) the total energy requirement is greater than 10^{55} erg by only one or two powers of 10, say 10^{57} erg, which is not much different from the energy output suggested by Burbidge and Hoyle.

The second possibility is that the objects have emerged from a powerful radio galaxy in the neighborhood of the Galaxy. The galaxy NGC 5128 is an immediate suggestion, because NGC 5128 is known to have undergone two major outbursts in the last few million years. Taking 10 Mpc as the characteristic distance in this case, the total energy requirement is increased to $\sim 10^{59}$ erg, equivalent to the rest mass energy of $\sim 10^5 M_{\odot}$. This also is consistent with what is thought to have been involved in the outbursts of NGC 5128.

If the objects come from the Galaxy no cases showing a Doppler blue shift are to be expected. If the objects come from NGC 5128 there is the possibility that some slowly moving objects still lie between the Galaxy and NGC 5128. These

would show a blue shift. Consider objects to have been emitted isotropically from NGC 5128 with speed v a time τ ago, and let D be the distance between NGC 5128 and the Galaxy. Evidently $v\tau/D$ is dimensionless. The fraction of objects showing blue-shift is $0.5 (1 - v\tau/D)$ if $v\tau/D < 1$ and is zero otherwise, and in the case $v\tau/D < 1$ the blue shifted objects are found in a solid angle

$$2\pi \left(1 - \left[1 - (v\tau/D)^2\right]^{\frac{1}{2}}\right)$$

centered on NGC 5128. For $v\tau/D$ small, approximately half of the objects have blue shifts but on the sky they are concentrated closely around NGC 5128, e.g., $v\tau/D = 0.1$ gives 45% blue shifts but the solid angle about NGC 5128 is only 0.031 steradian. As $v\tau/D$ increases, so does the solid angle but the blue-shifted fraction decreases, e.g., $v\tau/D = 0.6$ gives a solid angle of ~ 1.25 steradian but the fraction of blue-shifted objects has fallen to 20%.

Taking 10 Mpc as the characteristic distance, say for $v = c/2$, we require $\tau \approx 60$ million years for the time that has elapsed since the relevant explosion in NGC 5128. The distance D is rather uncertain, 4 Mpc is the current estimate. With these values we have $v\tau/D = 0.6$ for $v \approx 0.1 c$. Blue shifts of this amount in directions toward NGC 5128 could confirm this theory. Absence of blue shifts would go a long way toward disproving it, although it may be possible to increase τ/D sufficiently for only very small shifts to be permitted and there could be a paucity of slowly moving objects.

It is a point against objects from NGC 5128 that τ must be taken at least as great as ~ 30 million years if the objects are to appear approximately isotropic when viewed from the Galaxy. This is longer than the time which has elapsed since the first of the two known explosions in NGC 5128, assuming the latter to be given by dividing the dimension of the extended radio source around NGC 5128 by the velocity of light. Possibly the extended source is no longer

expanding at appreciable speed, in which case the elapsed time since the first outburst could be ~ 30 million years in consonance with the present requirement. Alternatively it seems possible that NGC 5128 has undergone a succession of explosions before the one which gave rise to the extended radio source which we see at present.

Finally, we notice that the characteristic distances given above, ~ 1 Mpc for objects from the Galaxy, ~ 10 Mpc for objects from NGC 5128, are so great that proper motions must be very small. For example, an object at 10 Mpc with transverse motion $0.1 c$ would have a proper motion less than $0.001''/\text{yr}$. Estimates by Luyten (1963) and by Jeffreys (1965) place the proper motion of 3C 273 at less than $0.01''/\text{yr}$ (Luyten) and less than $\sim 0.0025''$ (Jeffreys).

IV. SYNCHROTRON EMISSION BY 3C 273B

The recent remarkable observation by Dent (1965) of an increase in the intensity of 3C 273B, by about 40% at 8000 Mc/s over the past two and a half years, has an important bearing on the theories discussed in the two preceding sections, particularly on the cosmological theory. At 8000 Mc/s the emitting region cannot be much larger than a few parsecs, say $< \sim 3$ parsec. In the present section we shall attempt a discussion of the significance of this new datum, when taken together with the fact that the spectrum of 273B is flat over the range from 200 Mc/s up to 8000 Mc/s, and may indeed be flat over the much wider range from ~ 100 Mc/s up to $\sim 10^8$ Mc/s, i.e., up to $\sim 3 \mu$.

Suppose first we follow the conventional picture of the frequency spectrum being determined by the energy spectrum $d\gamma/\gamma^n$ of the electrons, the magnetic field H being constant, and the electron energy being γmc^2 . For a flat spectrum we require $n = 1$ over the relevant range of γ , this being determined by H and by the range of frequency over which we require the radio spectrum to be flat.

To avoid synchrotron self-absorption, which would destroy the flat spectrum, giving a $\nu^{2.5} d\nu$ law on a short time scale and a $\nu^2 d\nu$ law on a long time scale, it is necessary that the emission rate shall not approach the 'black body' value for a temperature given by setting kT equal to an appropriately chosen energy value for the emitting electrons. Let r be the radius. Then we require

$$4\pi r^2 \times \frac{2\pi \nu^2 \gamma mc^2}{c^2} > 6 \times 10^{33} \text{ erg sec}^{-1} (\text{c/s})^{-1} \quad (4)$$

in the cosmological theory. The observed radio intensity over the flat spectrum leads to an emission requirement of $6 \times 10^{33} \text{ erg sec}^{-1} (\text{c/s})^{-1}$ in this theory. In the local theory, on the other hand, it would be necessary to reduce the right hand side of (4) by 10^{-4} if the characteristic distance of the quasi-stellar objects is $\sim 10 \text{ Mpc}$, and by 10^{-6} if the characteristic distance is $\sim 1 \text{ Mpc}$. The effective kT in (4) is γmc^2 , where γ is related to ν by

$$\nu \approx 4 \times 10^6 \gamma^2 H, \quad (5)$$

with H in gauss.

Condition (4) becomes more severe as ν is reduced, that is to say higher values of γ are needed. setting $\nu \approx 2 \times 10^8 \text{ c/s}$, the lowest value for which the spectrum is known to be flat, taking $r = 10^{19} \text{ cm}$, gives $\gamma \geq \sim 2 \times 10^4$. Accepting the least permissible value, $\gamma = 2 \times 10^4$, and inserting in (5), together with $\nu = 2 \times 10^8 \text{ c/s}$, we obtain $H \approx 10^{-7} \text{ gauss}$, a very low value. Now the lifetime of an electron of energy $\gamma = 2 \times 10^4$ in a field of intensity 10^{-7} gauss is very long, $\sim 3 \times 10^{18} \text{ sec}$. This means that the rate of emission at all frequencies up to $\nu \approx 2 \times 10^8 \text{ c/s}$, i.e., $\sim 10^{42} \text{ erg sec}^{-1}$, can be only a fraction, $\sim 3 \times 10^{-19}$, of the energy of the whole reservoir of electrons. The latter therefore must have a total energy of $\sim 3 \times 10^{60} \text{ ergs}$.

The problem now arises as to how such a vast energy can be contained within a volume of a few cubic parsecs. Certainly not by a magnetic field as low as

10^{-7} gauss. The possibility arises, however, that the magnetic field is anchored to a cloud of ambient gas of total mass M_1 held in the gravitational field of a central object of mass M_2 . The gravitational energy of the cloud would be $\sim GM_1 M_2/r$ and for there to be any possibility of stability we must have

$$\frac{GM_1 M_2}{r} > 3 \times 10^{60} \text{ erg.} \quad (6)$$

Greenstein and Schmidt (1964) have estimated (on the cosmological theory) a total mass of the general order of $10^6 M_\odot$ for the gas cloud around 3C 273.

Inserting this value for M_1 , (6) gives $M_2 > 10^{13} M_\odot$, r being again taken as 10^{19} cm. This seems an impossible requirement. We arrive therefore at the conclusion that one or other of the following three possibilities represents the true situation.

- (i) the cosmological theory is incorrect,
- (ii) some process other than synchrotron radiation is responsible for the radio emission,
- (iii) a radically different model of the synchrotron radiation from that used above must be found.

On thermodynamic grounds (ii) does not seem plausible to us, since a similar argument to that given above must hold unless a process can be found that leads to a higher effective value of kT than is given by the energies of relativistic electrons, and this seems most unlikely. In the remainder of this section we shall discuss a different model for the synchrotron radiation that seems capable of fitting the data, except possibly the flatness of the spectrum in the infra-red.

The essential point in the following discussion is that the radio frequency spectrum is controlled by variations in the magnetic intensity rather than by the form of the electron energy distribution. First we show that the frequency

spectrum is flat if the following postulates are satisfied:

- (1) The electron distribution is 'optically thin'.
- (2) The magnetic field intensity H depends on distance r from the center of the object according to

$$H = H_0 (a/r)^n, \quad (7)$$

where H_0 , a , n are constants.

- (3) The electron energy spectrum is everywhere the same.
- (4) The energy density, W , of the electron distribution depends on r according to

$$W = W_0 (b/r)^m,$$

where W_0 , b , m are constants.

- (5) $n + m = 3$.

From (5) and (7) we have

$$\nu \approx 4 \times 10^6 \gamma^2 H_0 (a/r)^n. \quad (8)$$

The rate of emission by an individual electron of energy γmc^2 is

$$1.6 \times 10^{-15} \gamma^2 H^2, \quad (9)$$

and, using postulates (3) and (4) it is not hard to see that the total emission from all electrons of energy γmc^2 inside a sphere of radius r must be of the form

$$\text{constant} \int_0^r r^2 \left(\frac{a}{r}\right)^{2n} \left(\frac{b}{r}\right)^m dr = \frac{\text{constant}}{r^{2n+m-3}} + \text{constant}. \quad (10)$$

In view of postulate (1) this radiation escapes from the system.

Differentiating (10), we see that the energy emitted by the electrons of energy γmc^2 in the shell between r and $r + dr$ is of the form

$$(\text{constant}) \cdot \frac{dr}{r^{2n+m-2}}. \quad (11)$$

Differentiating (8), we have

$$dv = (\text{constant}) \times \frac{dr}{r^{n+1}}. \quad (12)$$

Eliminating dr between (11) and (12) it is seen that the energy radiated in the frequency range ν to $\nu + dv$ is of the form

$$(\text{constant}) \times \frac{dv}{r^{n+m-3}}, \quad (13)$$

which is flat if $n + m = 3$. Next, we notice that, if postulates (2) and (4) hold for large r , the spectrum (13) applies down to small ν , ν being related to r by (8). And if postulates (2) and (4) hold for small enough r the spectrum (13) applies up to large ν . It follows that provided $n + m = 3$ the spectrum from electrons of energy γmc^2 is then flat over the whole range of interest in ν . If this is true for any value of γ it is true for all values of γ , and hence for any electron spectrum.

On physical grounds two cases are of immediate interest:

(a) $n = 3$, $m = 0$, corresponding to a dipole magnetic field and a uniform electron energy distribution.

(b) $n = 2$, $m = 1$, corresponding to a 'pulled out' field and an electron energy distribution which falls off as $1/r$.

Case (a) corresponds to what might be expected if the system is not embedded in a cloud of ambient gas. Case (b) is what might be expected if an external cloud of ambient gas plays an important role in anchoring the system. We discuss these cases in turn.

(a) The Dipole Case

To show this case is applicable to 3C 273B it is necessary that two conditions be reconciled:

- (1) That (4) is satisfied down to $\nu \approx 200$ Mc/s, otherwise postulate (1) could not hold.

- (11) The energy density of the electron distribution must not exceed the energy density of the magnetic field otherwise the electrons would explode outward.

It will now be shown that these two conditions can just be met.

The crucial difference between the present discussion and that given at the beginning of this section is that the value of r to be inserted in (4) is not independent of ν . If we use the work of Dent to show that r must not exceed 10^{19} cm we must therefore be careful to use the appropriate ν , $\sim 10^4$ Mc/s. Inserting these values in (4) gives

$$\gamma mc^2 > \sim 10^{-5} \text{ erg}, \quad (14)$$

a trivial requirement. For other values of ν it is necessary to take account of the fact that ν , γ , and r are related by (8). The simplest procedure is to use (8) to eliminate r from (4) giving a condition of the form

$$\gamma^{n+4} > (\text{constant}) \times \nu^{2(1-n)}. \quad (15)$$

The constant in (15) can immediately be determined from (14) at $\nu = 10^4$ Mc/s.

Evidently the appropriate condition is

$$\gamma mc^2 > \sim 10^{-5} \left(\frac{10^{10}}{\nu} \right)^{\frac{2(n-1)}{n+4}} \text{ erg}, \quad (16)$$

ν being in c/s. This condition must hold down to $\nu = 200$ Mc/s. In the dipole case $n = 3$, and (16) gives $\gamma mc^2 > \sim 10^{-4}$ erg, i.e., $\gamma mc^2 > \sim 0.1$ Bev, an entirely reasonable requirement.

We shall now show that requirement (11) is also satisfied provided γ is set as low as we are permitted by requirement (1). This is the lower limit given by (16), $\gamma mc^2 \approx 0.1$ Bev, γ of order 10^2 . In the following we use $\gamma = 10^2$, and we carry through the calculation for $\nu = 200$ Mc/s, because the energy requirements are most severe at the lowest frequencies.

To obtain $\nu = 200$ Mc/s for $\gamma = 10^2$ we require $H \approx 5 \times 10^{-3}$ gauss. The question arises as to what value of r should be associated with this magnetic intensity. From (8) with $n = 3$ we have $\nu \propto r^{-3}$ when γ is fixed. In this model we are adopting $r = 10^{19}$ cm when $\nu = 10^4$ Mc/s, so for $\nu = 200$ Mc/s we must take $r = (50)^{1/3} \times 10^{19}$ cm, and this is to be associated with $H \approx 5 \times 10^{-3}$ gauss. These values determine the constant of proportionality for the dipole field, $H \propto r^{-3}$, giving

$$H \approx 10 r^{-3} \text{ gauss}, \quad (17)$$

in which r is in parsec.

The lifetime of an electron with $\gamma = 10^2$ in a field of intensity 5×10^{-3} gauss is $\sim 2 \times 10^{11}$ sec. Since the total emission of 3C 273B up to $\nu = 200$ Mc/s is $\sim 10^{42}$ erg sec $^{-1}$, in the cosmological theory, we evidently require an electron reservoir with a total energy of $\sim 2 \times 10^{53}$ erg. This is very much less than was required when the object was taken as having the same radius, $r \approx 10^{19}$ cm, at all values of ν , in the calculation given at the beginning of this section. The large differences between the present estimate and the value of order 10^{60} erg obtained in the first investigation demonstrates the sensitivity of the situation to the assumed model. The electron energy density is now further reduced because the radius is $(50)^{1/3} \times 10^{19}$ cm, instead of 10^{19} cm. The total volume of the electron reservoir is therefore $\sim 2 \times 10^{59}$ cm 3 , so that the electron energy density is $\sim 10^{-6}$ erg cm $^{-3}$, the same as the energy density of the field with strength 5×10^{-3} gauss. Hence requirement (ii) is just satisfied.

It is necessary to emphasize the sensitivity of this result to the chosen value of γ . Adjusting the magnetic intensity to give a specified frequency, 200 Mc/s in the above discussion, the field varies with γ as γ^{-2} , and the lifetime of the electrons varies as γ^3 . Hence the total energy of the magnetic field varies as γ^{-4} and the total energy of the electron reservoir (needed to

give the observed emission) varies as γ^3 , so that the ratio of the total magnetic field to the electron reservoir varies as γ^{-7} . The value $\gamma = 10^2$ used above leads to a ratio of order unity. Changing γ by only a factor 2 would change the ratio a hundredfold.

Following from this, it is clear that the whole calculation is enormously sensitive to the value chosen for r at 10^4 Mc/s. Avoidance of synchrotron self-absorption at any specified frequency sets the lower limit for γ , of the form $\gamma > (\text{constant}) r^{-2}$, as can be seen from (4). Adjusting the magnetic intensity to give the specified frequency we again require $H \propto \gamma^{-2} \propto r^4$. The total magnetic energy therefore varies as r^{11} , a factor r^3 entering for the volume. The lifetime of an individual electron again varies as γ^3 , and so does the total energy of the electron reservoir necessary to give the observed emission. Hence the total energy of the electron reservoir varies as r^{-6} , and the ratio of the total magnetic energy to the total electron energy varies as r^{17} . In the present model the radius at $\nu = 200$ Mc/s is greater than it was in the calculation given at the beginning of this section by the factor $(50)^{1/3}$. When raised to the 17^{th} power this increase of r alters the energy ratio by $\sim 10^{10}$, which explains why the magnetic field is just able to restrain the electron reservoir from exploding, whereas in our first calculation it was utterly unable to do so.

The sensitivity of these factors makes it clear that one must be cautious in asserting that the new work of Dent (1965) makes a synchrotron explanation of the radio emission of 3C 273B impossible - this on the cosmological theory. Small changes in the basic parameters, particularly the radius, greatly affect the outcome of the calculations.

(b) The Case $n = 2, m = 1$

Inserting $n = 2, \nu = 200$ Mc/s in (16), we see that the condition set by

the absence of synchrotron self-absorption is $\gamma mc^2 > \sim 5 \times 10^{-5}$ erg, a somewhat weaker requirement than before, again permitting $\gamma \approx 10^2$. With this value of γ we again require $H \approx 5 \times 10^{-3}$ gauss to give emission at $\nu = 200$ Mc/s. We again note that our calculations of energy requirements are carried out at the lowest frequencies because the requirements are more severe at low frequencies than at high frequencies. Because H is now proportional to r^{-2} we have

$$H \approx r^{-2} \text{ gauss} \quad (18)$$

in place of (17).

The balance of the electron energy requirement and of the total magnetic energy are again much the same as before, and the same remarks apply to the sensitivity of the ratio of these quantities. In the present case, however, we can contemplate that the total magnetic energy falls below the total energy of the electron reservoir, because now we can call on the inertia of an ambient gas cloud to restrain the electron distribution. The latter is again of order 10^{53} erg, and in the notation used at the beginning of this section we only require

$$G \frac{M_1 M_2}{r} > \sim 10^{53} \text{ erg} \quad (19)$$

instead of the much more severe condition in (3). Again using $10^6 M_\odot$ for M_1 , we only require $M_2 > \sim 10^7 M_\odot$, which can certainly be satisfied.

To understand the choice $m = 1$, it is possible to argue that the electron distribution has built up to the maximum that can be restrained by the ambient gas - i.e., that $G M_1 M_2 / r$ is close to the lower limit set by (19). Any further build up would lead to high energy electrons escaping into space. Now the simplest assumption concerning the ambient gas is that the density is uniform, in which case the gravitational potential energy per unit volume falls as r^{-1} , provided the interior mass is greater than the exterior mass. The

restraining effect of the ambient gas would fall off as r^{-1} in such a picture, and the maximum permitted electron density would also fall as r^{-1} , i.e., $m = 1$.

The above discussion seems to us to show that a synchrotron origin for the flat spectrum of 3C 273 is certainly possible in the cosmological theory, at any rate for frequencies up to 10^4 Mc/s. A question does arise, however, as to how far the spectrum can continue to be flat. We shall examine this question, by way of concluding the present section.

Synchrotron Emission at High Frequencies

Our model requires the frequency spectrum to be controlled by the variation of H with r , not by the electron energy spectrum. It was this change from the usual picture, in which H is taken constant and the frequency spectrum is controlled by the electron distribution, that permitted the effective radius r of the emitting region to increase as the frequency ν was lowered. For consistency, we must continue to adopt the same model at frequencies upward of 10^4 Mc/s. Quantitatively, this means that we must continue to calculate for the same values of γ , viz $\gamma \approx 10^2$, that led to a consistent situation at the lowest frequencies. A difficulty now arises that the electron lifetimes become very short. At $\nu = 200$ Mc/s, we had $r \approx (50)^{1/3} \times 10^{19}$ cm, and we obtained an electron lifetime of $\sim 2 \times 10^{11}$ sec. Taking the case $n = 2$, $m = 1$ as being more favorable than the dipole case at very high frequencies, we have $\nu \propto H \propto r^{-2}$ for fixed γ . Numerically,

$$\nu \approx 2 \times 10^8 \left(\frac{50^{1/3} \cdot 10^{19}}{r} \right)^2 \approx \frac{3 \cdot 10^{47}}{r^2} \text{ c/s} \quad (20)$$

for r in cm. The lifetime τ of an electron of fixed γ varies with H as H^{-2} , i.e., as r^4 . Numerically,

$$\tau \approx 2 \times 10^{11} \left(\frac{r}{50^{1/3} \times 10^{19}} \right)^4 \approx 10^{28} \nu^{-2} \text{ sec}, \quad (21)$$

for ν in c/s. At $\nu = 10^{10}$ c/s, $\tau \approx 10^8$ sec. Although short, this is still permissible. However, raising ν to 10^{14} c/s, the highest frequency to which the flat spectrum of 3C 273B may well extend, leads to the absurdly short lifetime of ~ 1 sec.

A reasonable condition on τ is that τ must not be appreciably less than r/c . Setting

$$\tau = \frac{r}{c} \approx 10^{28} \nu^{-2}, \quad (22)$$

we can eliminate r between (20) and (22), giving $\nu \approx 10^{10}$ c/s, and this is the highest frequency to which the flat spectrum can reasonably be expected to extend. The model evidently runs into difficulty over the extension of the flat spectrum to frequencies higher than this. A way around the difficulty would be to go back to the uniform field case at frequencies above 10^{10} c/s - i.e., to argue that H is approximately constant for r less than $\sim 10^{19}$ cm. We could rely on $H \propto r^{-2}$, $r \geq \sim 10^{19}$ cm to explain the spectrum for $\nu < \sim 10^{10}$ c/s, along the lines discussed above, and on $H = \text{constant}$, $r < \sim 10^{19}$ cm with an electron energy spectrum dN/dV to explain the flat spectrum for $\nu > \sim 10^{10}$ c/s. This is a somewhat artificial device but the possibility cannot be excluded.

We have concentrated completely in this section on attempting to find a model which will account for the radio frequency observations of Dent, and have not tried to relate this to the models for the optical object which have been proposed by Greenstein and Schmidt (1964), Oke (1965), and Shklovsky (1964). It may be possible to reconcile this type of model with that required to explain the line strengths and continuum in the optical region, though the need for a magnetic field varying with distance from the center in a volume which probably overlaps with that required to explain the line emission may require some revision of these models.

IV. QUANTITATIVE CONSIDERATIONS IN THE LOCAL THEORY

The self-absorption condition (4) is very much weakened in the local theory. Write x for the factor by which the distance of 3C 273 is reduced. Then a factor x^{-2} must be inserted on the right hand side of (4), and this is $\sim 10^{-4}$ if the characteristic distance of the quasi-stellar objects is ~ 10 Mpc and is $\sim 10^{-6}$ if the characteristic distance is ~ 1 Mpc. The condition for lack of self-absorption ceases then to be of serious account in building a model of the synchrotron emission of 3C 273B.

It would be possible to return to the usual theory, with H constant and the frequency spectrum depending on the form of the electron energy distribution. The latter would have to be of the form $d\gamma/\gamma$. In our view, however, the general model developed in the preceding section, with the frequency spectrum determined by the variation of H with r , gives so convenient an explanation of the flat spectrum that we prefer to adhere to it even though there is now no strict necessity to do so. This model is very plausible for a compact object in which H must be expected to vary with the distance r from the center.

We proceed by increasing the important γ values by the factor y , $y > 1$, and by reducing r by the factor z , $z > 1$, i.e., γ is $\sim 10^2 y$. From what was said in the preceding section concerning the dependence on γ and on r of the ratio of the total magnetic energy to the total energy of the electron reservoir, it is clear that this ratio will be much reduced below unity if y , z are appreciably greater than unity. However, this is not a serious matter provided the electron distribution is restrained from explosion by the inertia of an ambient gas cloud. We then require

$$\frac{GM_1 M_2}{r} > \text{Electron Reservoir,} \quad (23)$$

and we consider the case $n = 2$, $m = 1$, on the same basis as before. Electron lifetimes at fixed ν are increased by y^3 , i.e., by y^3 . This increases the electron reservoir by y^3 . On the other hand the electron reservoir is reduced by x^{-2} because the required emission is less by this factor. Remembering that r is reduced by z , it is clear therefore that former values for r , and for the electron reservoir, can be inserted in (23) provided the factor $x^{-2} y^3 z^{-1}$ is included on the right hand side. Hence the condition that the electron distribution does not explode outwards is given by using the values $r \approx (50)^{1/3} \times 10^{19}$ cm, $\sim 10^{53}$ erg for the electron reservoir, leading immediately to

$$M_1 M_2 > \sim 10^{13} x^{-2} y^3 z^{-1} \quad (24)$$

in which M_1, M_2 are in M_\odot . Formerly we had $x = y = z = 1$, $M_1 \approx 10^6 M_\odot$, $M_2 > 10^7 M_\odot$. The estimate $M_1 \approx 10^6 M_\odot$ was taken from the work of Greenstein and Schmidt (1964) in which 3C 273 was assumed to be at a cosmological distance. This estimate must be reduced in the local theory. We defer a discussion of the values that M_1 and M_2 might have in the local theory to a later stage in the argument. Our immediate purpose is to see whether the difficulty arrived at in the preceding section, that the frequency spectrum cannot be maintained flat to ν much above 10^{10} c/s, can be overcome in the local theory.

Equation (20) and (22) must now be changed, (20) because at fixed ν the distance r is reduced by z , so that

$$\nu \approx \frac{3 \times 10^{47}}{r^2} z^{-2} \quad (25)$$

and (22) because the electron lifetimes are now increased by y^3 . Thus

$$\frac{r}{c} \approx 10^{28} \nu^{-2} y^3. \quad (26)$$

Eliminating r between (25) and (26) again gives the highest frequency up to which the spectrum can be flat, viz

$$\nu \approx 10^{10} y^2 z^{2/3} \text{ c/s.} \quad (27)$$

For y, z large enough the frequency given by (27) can be increased to the value of order 10^{14} c/s indicated by observation. However, such choices for y, z must be checked against (24) that they lead to permissible values for M_1, M_2 . Also the self-absorption must be kept negligible. The latter is easily verified.

Condition (4) is altered by the factor $x^{-2} y z^2$ placed on the right hand side, so we require

$$x^{-2} y z^2 < 1 \quad (28)$$

for (4) to remain satisfied. It would be reasonable to set $y \approx 10$, raising the important electron energies to ~ 1 Bev. With z also of order 10, (28) is satisfied both when the characteristic distance of the quasi-stellar objects is 10 Mpc and when the characteristic distance is 1 Mpc, giving $x^{-2} \approx 10^{-4}$ and 10^{-6} respectively.

With $y = z = 10$, the spectrum is flat up to $\nu \approx 10^{13}$ c/s, perhaps not quite high enough but certainly not in gross disagreement with observation. Condition (24) remains to be satisfied, however. When $x^{-2} \approx 10^{-4}$, it would not be unreasonable for the central mass M_2 to be set as high as $10^6 M_\odot$, in which case (24) requires $M_1 > 10^5 M_\odot$. Although this condition is satisfied by the estimate of Greenstein and Schmidt, this estimate must now be reduced. A mass of ambient gas as large as this cannot be present within a radial distance of $\sim (50)^{1/3} \times 10^{19} z^{-1}$ ($\approx 3 \times 10^{18}$ cm for $z \approx 10$) if the gas is largely ionized, because self-absorption due to free-free transitions would occur, leading to a $\nu^2 d\nu$ law at the low frequencies instead of the flat spectrum. If the gas were mostly un-ionized this difficulty would not arise of course. Even so, the necessary mass seems high.

The case $x^{-2} \approx 10^{-6}$, corresponding to objects at a characteristic distance of ~ 1 Mpc, is somewhat more favorable. With $y = z = 10$, (24) then gives

$M_1 M_2 > \sim 10^9$. If M_2 can still be taken as high as $10^6 M_\odot$ the condition $M_1 > \sim 10^3 M_\odot$ could perhaps be satisfied.

We conclude this aspect of our discussion with the conclusion that extension of the flat frequency spectrum to 10^{14} c/s, or even to 10^{13} c/s, still presents difficulties, although the difficulties are not so severe in the local theory as they are in the cosmological theory. For example, the choice $z \approx 30$, $y = 3$ gives a flat spectrum up to $\sim 10^{12}$ c/s and (24) only requires $M_1 M_2 > \sim 10^7$ in the case $x \approx 10^{-6}$.

V. : THE RED-SHIFT APPARENT MAGNITUDE RELATION IN COSMOLOGY AND IN THE LOCAL THEORY

In this section we shall be concerned with the red-shift - magnitude relation to be expected in the local theory, assuming the quasi-stellar objects to be isotropically distributed with respect to the Galaxy. First, however, we outline the derivation of this relation in orthodox cosmology. Taking the cosmical constant as zero, Einstein's equations are, in the usual notation

$$\frac{\dot{R}^2 + k}{R^2} = \frac{8\pi G\rho}{3}, \quad (29)$$

$$\frac{2\dot{R}}{R} + \frac{\dot{R}^2 + k}{R^2} = 0. \quad (30)$$

The line element is

$$ds^2 = dt^2 - R^2(t) \left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2 \right], \quad (31)$$

in which t is the time, with a unit such that $c = 1$. The apparent bolometric magnitude of an object of fixed intrinsic emission is related to the red-shift and to the coordinate distance r by

$$\text{Apparent Luminosity} = \frac{\text{Constant}}{R_0^2 r^2 (1+z)^2}, \quad (32)$$

where

$$1 + z = 1 + \frac{\Delta\lambda}{\lambda} = \frac{R_0}{R(t)}, \quad (33)$$

in which R_0 is the value of R at reception and $R(t)$ is the value at emission. All objects have effectively the same moment of reception but different objects in general have different values of $R(t)$. The density ρ is related to R by

$$\rho = \rho_0 \left(\frac{R_0}{R} \right)^3, \quad (34)$$

where ρ_0 is the cosmological density at the present moment of reception.

It is always possible to change the unit of length, changing t , R by a scale factor, but keeping the r coordinate unchanged. Choose a scale factor such that R_0 satisfies

$$\frac{8\pi}{3} G \rho_0 R_0^3 = 1, \quad (35)$$

so that (29) takes the simplified form

$$\dot{R}^2 + k = \frac{1}{R}. \quad (36)$$

The constant k can be 0, or ± 1 . The work of Schmidt (1965) on the redshifts of quasi-stellar radio sources has suggested that, if the sources are cosmological, the case $k = 1$ may give the most suitable model. We then have

$$\dot{R}^2 = \frac{1 - R}{R}. \quad (37)$$

It remains to relate r with $z = \Delta\lambda/\lambda$, so that the right hand side of (32) can be expressed as a function of z . This step is achieved by noticing that $ds = 0$ along a light track, giving

$$\frac{dr}{1-r^2} = - \frac{dt}{R}, \quad k = 1. \quad (38)$$

Using (37),

$$\int_0^r \frac{dr}{1-r^2} = \int_R^{R_0} \frac{dR}{R^{\frac{1}{2}} \sqrt{1-R}} \quad , \quad (39)$$

which integrates to

$$\sin^{-1} r = 2 [\sin^{-1} R_0^{\frac{1}{2}} - \sin^{-1} R^{\frac{1}{2}}]. \quad (40)$$

Here we use the fact that $r = 0$ for the observer. The limits of the integrals in (39) are arranged so that both are positive. It may be noted that r is associated with R and 0 with R_0 . This inversion is brought about through the minus sign in (38). From (40),

$$r = 2 \left[R_0^{\frac{1}{2}} \sqrt{1-R_0} (1-2R) - R^{\frac{1}{2}} \sqrt{1-R} (1-2R_0) \right]. \quad (41)$$

Substituting $R = R_0/(1+z)$ gives the required relation between r and $1+z$.

However, in general the apparent luminosity depends on R_0 as well as on $1+z$, not merely as a multiplicative factor but in a complicated way. That is to say, the right hand side of (32) cannot in general be reduced to a simple product $f(R_0) g(1+z)$, although in a certain special case it can be so reduced.

A parameter q_0 is introduced through the definition

$$\left(\frac{\ddot{R}}{R} \right)_0 = -q \left(\frac{\dot{R}}{R} \right)_0^2. \quad (42)$$

Applying (30) for $k = 1$ and for the present moment,

$$\left(\frac{\ddot{R}}{R} \right)_0 = -\frac{1}{2} \left(\frac{\dot{R}^2 + 1}{R^2} \right)_0, \quad (43)$$

so that

$$(2q - 1) (\dot{R}^2)_0 = 1. \quad (44)$$

Should q happen to be unity at the present moment, $(\dot{R}^2)_0 = 1$. Hence, from

(37), $R_0 = 0.5$. Substituting this value in (41), we obtain the simple result

$$r = 1 - 2R = \frac{z}{1+z}, \quad (45)$$

and (32) gives

$$\text{Apparent Luminosity} = \frac{\text{constant}}{z^2} \quad (46)$$

This is the case which has been considered to give the best fit to the red-shift data of Schmidt (1965). We again note that (46) applies to the bolometric luminosity. If the contribution of a fixed frequency range, or a fixed wave-length range, is required, appropriate factors in $1 + z$ have to be included on the right hand side of (46), $1 + z$ for a fixed frequency range, $(1 + z)^{-1}$ for a fixed wavelength range.

We proceed now to compare (46) with what is to be expected in the local theory. Suppose a number of objects to be emitted from the observer's position at time $t = 0$. Again choose the time unit so that $c = 1$. Light emitted at time t from an object moving at speed v reaches the observer at time $t(1 + v)$. For observation at a particular moment of time we therefore require

$$t = \frac{\text{constant}}{1 + v}, \quad (47)$$

and the distance of the object at emission is

$$vt = \text{constant} \times \frac{v}{1 + v}. \quad (48)$$

Hence for objects all with the same intrinsic emission we have

$$\text{Apparent Luminosity} = \frac{\text{constant}}{(1 + z)^2} \times \left(\frac{1 + v}{v} \right)^2. \quad (49)$$

The factor $(1 + z)^{-2}$ arises from the red-shift and number effects, which apply here the same as in the cosmological theory. The factor $[(1 + v)/v]^2$ is just the inverse square of the distance at the moment of emission - we calculate now for simple Euclidean space. In flat space we have the following relation between z and v ,

$$1 + z = \left(\frac{1 + v}{1 - v} \right)^{\frac{1}{2}}. \quad (50)$$

Eliminating v between (49) and (50),

$$\text{Apparent Luminosity} = \frac{\text{constant}}{z^2} \cdot \frac{(1+z)^2}{(1+z/2)^2} \quad (51)$$

The local theory differs from the cosmological theory with $k = 1$, $q_0 = 1$, in that the apparent luminosity is increased by the extra factor $(1+z)^2/(1+z/2)^2$. Schmidt (1965) discusses the intrinsic luminosities which 9 quasi-stellar sources must have (in the cosmological theory, $q_0 = 1$) in order to explain the apparent luminosities. A similar calculation in the local theory would yield intrinsic values less than those calculated by Schmidt, by the factor $(1+z/2)^2/(1+z)^2$. This has the effect of bringing the intrinsic values very close together for all of the sources except 3C 273, which then stands out as about 5 times brighter than the others.

The present considerations apply directly to the local theory of Terrell, in which the quasi-stellar objects are considered to have been expelled by our own Galaxy. For the case of objects expelled from NGC 5128 the situation is more complicated, however, because our point of observation is offset from the point of ejection. The relation (51) should apply to a good approximation for objects with $v \rightarrow c$, since such objects will be most distant, e.g., 10 Mpc or more, and the offset effect is not then very important. But for slowly moving objects, at distances comparable to that of NGC 5128 itself, variations by a moderate factor from (51) are to be expected. It is possible that 3C 273 is such a case, since for this object $z = 0.158$.

If the objects are all of the same linear size, their apparent angular diameters are inversely proportional to their distances at the moment of emission - i.e., proportional to $(1+v)/v$, to

$$\frac{1}{z} \propto \frac{(1+z)^2}{1+z/2} \quad (51)$$

In the cosmological theory angular diameters for objects of fixed linear size are inversely proportional to $r R(t)$. For the case $q_0 = 1$, the factor r is given simply in terms of z by (45). Also $R(t) = R_0(1 + z)$, so that in this theory angular diameters are proportional to

$$\frac{(1 + z)^2}{z}. \quad (52)$$

Angular diameters are smaller in the local theory by the factor $(1 + z/2)^{-1}$.

The problem of angular diameters is of course of critical importance. No object showing short term fluctuations of radio emission can have more than a very small angular diameter in the cosmological theory. Taking 10 parsecs for the radius of 3C 273B, the angular diameter should be about $0.008''$. This is much less than values of $\sim 0.5''$ given by Hazard, Mackey, and Shimmins (1963) and by Scheuer (1965). These determinations would rule out the cosmological theory if it could be confirmed that a diameter of order $0.5''$ referred to a single compact object. Also Scheuer gives an experimental profile of $\sim 0.3''$ which is so close to $0.5''$ that it seems permissible to regard the present situation as uncertain. We simply note that the angular diameter for an object of radius 1 parsec at a distance of 1 Mpc is $\sim 0.4''$, so that appreciable angular diameters are to be expected if the quasi-stellar objects are very local. Values of $\sim 0.04''$ would correspond to the case in which the objects are at characteristic distances of ~ 10 Mpc.

VI. CONCLUSIONS

This paper has been concerned with the possible origins of the star-like objects which are neither stars nor normal galaxies. Of the $\sim 10^5$ objects which are probably present down to 19^m spectra in which Doppler shifts can be measured have so far been obtained for fourteen objects, and redshifts have

been obtained in all of these. The situation as to their origin is rather similar to that which existed 50 years ago when the spiral nebulae were also a great mystery. About the same number of Doppler shifts had been measured, largely by Slipher, and there was considerable confusion as to whether they were of galactic or extragalactic origin. As is well known conclusive proof of their extragalactic nature came in the next decade.

With the discovery of the redshifts of the quasi-stellar radio sources, the most natural theory was to assume that these were also objects at cosmological distances and with the exception of the proposal by Terrell this is what has been generally assumed. However, in this paper we have attempted to discuss the physical nature of the objects, assuming either that they are at cosmological distances or that they are extragalactic but local at distances typically of 1 - 10 Mpc.

There are a number of observational programs which may eventually indicate which of these hypotheses is correct. In concluding we shall list some of these.

(1) The model that we have proposed to account for the form of the spectrum and the variability at high frequency observed by Dent in 3C 273B is just able to give rise to a flat spectrum out to about 10^4 Mc/s if the object is at a cosmological distance. Detailed observations out into the infrared will enable this model to be tested further.

(2) As has been emphasized by many authors detailed and accurate studies of 3C 273 and other star-like objects in all possible frequency ranges are badly needed to determine the time scales over which they vary.

(3) The angular diameter of the radio source 3C 273B is obviously of critical importance in deciding whether it is a very distant object. This question has been discussed at the end of Section IV. If the object can be proved to have

an angular diameter $\sim 0''.5$ and also is variable indicating a dimension of a few light years and is truly a single compact object then it must be local.

(4) Identification of more bright star-like objects may enable a significant test to be made as to whether or not they are associated with clusters of galaxies.

(5) If the objects are in general at cosmological distances then the bulk of them fainter than 16^m should have redshifts $z > 1$. If this is found to be the case the local origin hypothesis will not be disproved. However, if many of the faint star-like objects are found to have small redshifts $z \leq 0.1$ the model proposed by Sandage cannot be retained. The compact object discovered by Arp (1965) which is distinguishable from a star on a good direct plate has an apparent magnitude of 17.9^m and $z = 0.004$. On the local hypothesis this would probably be an object ejected from the Galaxy.

(6) The detection of objects with blue shifts would establish the correctness of the local explanation of such objects. On the picture described here NGC 5128 is a probable source, while some may come from our own Galaxy. We should not expect blue shifts from objects of galactic origin, but as was discussed in Section III a search for such objects bearing in mind that they may have come from NGC 5128 is urgently required. Since this is a southern galaxy ($\alpha = 13^h 22^m.4$, $\delta = -42^\circ 46'$ (1950)) searches in its vicinity must be carried out from the Southern Hemisphere.

(7) On the local theory depending on the time which has elapsed since objects were ejected from NGC 5128, we shall expect to see some asymmetry in the distribution of the objects on the sky. While it may be difficult to detect such asymmetry by optical methods, it is important that the distribution of the radio sources of small angular diameter over the sky be investigated, since on the local theory that fraction of the radio sources associated with quasi-stellar objects are local.

(8) If the ejection of coherent objects from the nuclei of galaxies is commonplace it may be possible to detect them about galaxies such as M 82 in which explosive events are known to have taken place comparatively recently. It is interesting that an optical identification of a quasi-stellar object of 19^m with a radio source very close to NGC 4651 (which was originally identified as the source) has recently been made (Sandage, Veron, and Wyndham 1965). As these authors have pointed out, NGC 4651 has a very peculiar jet-like structure and on the local hypothesis the 19^m object has been ejected from that galaxy.

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